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Lambert, Kant and solidity: a matter of method

1 Introduction: a matter of method

Ernst Cassirer pointed out that Lambert had introduced a »peculiar and new point of view in the problem of knowledge«, that is, the idea that we can take empirical concepts and then, without questioning their »psychological origin«, discover in them »certain universal relations« and thus set the basis for a priori knowledge concerning these concepts.[1] As Lambert puts it, experience »provides at best an occasion [*Veranlassung*], in order to see whether and how far one can prove a priori«.[2] Gereon Wolters has focused on this topic in his seminal study on Lambert's theory of scientific knowledge, maintaining that »as far as I can see, Lambert is the first in the history of exact sciences and their methodology to establish the *program of a protophysical basis*«.[3] This program concerns the formulation of definitions and/or postulates concerning the unities of measure in the exact sciences (e.g. extension, duration, mass), leading from the basic sensations corresponding to these magnitudes to the first scientific propositions. For example, Lambert asks »whether the first propositions of mechanics can be proved [*erweisen*] necessarily and a priori, as Euclid has done with regards to the geometrical propositions.[4] This investigation takes place in a section of mechanics that Lambert calls »Dynamics«, starting from the concepts of »solidity« and »force«, and it provides an opportunity to test the limits of Locke's empiricist methodology. »Solidity«, listed by Lambert among the *Grundbegriffe* of science, had been already considered by Locke as a simple idea derived from experience. While recognizing the importance of Locke's analysis, Lambert objected that the latter »had looked for the simple concepts, but he missed

1 Ernst Cassirer: Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit. Zweiter Band. In: ders.: Gesammelte Werke. Hg. von Birgit Recki. 26 Bde. Hamburg 1998–2009, Bd. 3, p. 453.

2 Johann Heinrich Lambert: Gedanken über die Grundlehren des Gleichgewichts und der Bewegung. In: ders.: Beyträge zum Gebrauche der Mathematik und deren Anwendung. Bd. 2.2. Berlin 1770, § 6. On the a priori and a posteriori investigation concerning propositions and properties see Johann Heinrich Lambert: Neues Organon oder Gedanken über die Erforschung und Bezeichnung des Wahren und dessen Unterscheidung von Irrthum und Schein. Leipzig 1764 (LPS 1), Dianoiologie, §§ 634ff.

3 Gereon Wolters: Basis und Deduktion. Studien zur Entstehung und Bedeutung der Theorie der axiomatischen Methode bei J. H. Lambert. Berlin, New York 1980, p. 85.

4 Lambert: Gedanken über die Grundlehren (see fn. 2), § 1.

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the application of the method of establishing doctrinal systems [*Lehrgebäude*]«.[5] Locke »proceeds entirely a posteriori« with his »anatomy of concepts«,[6] and thus misses the a priori moment in the foundations of empirical science, which can be conceived following the model of geometry:

> Es scheint ihm an der Methode, oder wenigstens an dem Einfalle gefehlet zu haben, das was die Meßkünstler in Absicht auf den Raum gethan hatten, in Absicht auf die übrigen einfachen ebenfalls zu versuchen.[7]

italics

Now, it is well known that Lambert's foundational attempt provided a substantial inspiration for Kant's analogous investigation on the possibility of a metaphysics of nature at the time of their correspondence, between 1765 and 1771, and beyond. Kant himself would recognize the authority of Lambert for the »claims subsequently presented in the *Kritik der reinen Vernunft* in their whole context«.[8] Alison Laywine has convincingly argued that Lambert's conception of postulates and their use in the foundations of natural science may have been a model for Kant's transcendental analytics.[9] But if we take the cue from the concepts of solidity and force it makes all the more sense to consider Kant's own account of »pure physics« in the *Metaphysische Anfangsgründe der Naturwissenschaft* (MAN), published in 1786, as a late assessment of the issues formulated by Lambert. Starting from a puzzling reference to Lambert, I will compare the latter's and Kant's accounts of solidity as alternative methodological solutions to the same foundational problem concerning concepts and principles of natural science.

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5 Johann Heinrich Lambert: Anlage zu Architectonic oder Theorie des Einfachen und des Ersten in der philosophischen und mathematischen Erkenntniß. Riga 1771, Bd. 1 (LPS 3), § 14.

6 Lambert: Neues Organon (see fn. 2), Alethiologie, § 29.

7 Lambert: Anlage zu Architectonic (see fn. 5), § 10 (italics mine).

8 AA X, p. 278. Indeed, Kant was planning to dedicate the Critique to Lambert. See Immanuel Kant: Reflexion 5024, AA XVIII, p. 64: »(zur dedication.) sie haben mich mit ihren Zuschriften beehrt. Die Bemühung, auf Ihr Verlangen einen Begrif von der Methode der reinen Philosophie zu geben, hat eine Reihe von Betrachtungen veranlaßt, den mir noch dunkel liegenden Begrif zu entwikeln, und, indem die Aussichten sich mit dem Fortschritt erweiterten, die Antworten einem unaufhorlichen Aufschub ausgesetzt. Diese Schrift kann statt einer Antwort dienen, was den speculativen Theil betrift. Da sie ihren Aufforderungen und Winken zuzuschreiben ist, so wünschete ich, daß die Ihnen ganz angehörete durch die Bemühung, sie in Ihre Bearbeitung zu nehmen«.

9 Alison Laywine: Kant and Lambert on the geometrical postulates in the reform of metaphysics. In: Mary Domski (ed.): Discourse on a new method. Reinvigorating the marriage of history and philosophy of science. Chicago 2010, pp. 113–133.

2 ~~##Titel ergänzen##~~ Kant on Lambert and solidity

Kant's critical reference to Lambert's account of solidity appears in the *Remark* to *Theorem* 1 of the »Dynamics« chapter of the *Metaphysische Anfangsgründe*.

> Lambert und andere nannten die Eigenschaft der Materie, da sie einen Raum erfüllt, die *Solidität* (ein ziemlich vieldeutiger Ausdruck), und wollen, man müsse sie an jedem Dinge, *was existiert* (Substanz), annehmen, wenigstens in der äußeren Sinnenwelt. Nach ihren Begriffen müßte die Anwesenheit von etwas *Reellem* im Raume, diesen Wiederstand schon durch seinen Begriff, mithin nach dem Satze des Widerspruchs bei sich führen, und es machen, daß nichts anderes in dem Raume der Anwesenheit eines solchen Dinges zugleich sein könne. Allein der Satz des Widerspruchs treibt keine Materie zurück, welche anrückt, um in einen Raum einzudringen, in welchem eine andere anzutreffen ist. Nur alsdann, wenn ich dem, was einen Raum einnimmt, eine Kraft beilege, alles äußere Beweglich, welches sich annähert, zurück zu treiben, verstehe ich, wie es einen Widerspruch enthalte, daß in dem Raum, den ein Ding einnimmt, noch ein anderes von derselben Art eindringe.[10]

The concluding sentence refers to Kant's own alternative view, exposed in the theorem. Rather than to »solidity« (an expression that he considers suitable for »rigid« bodies),[11] Kant refers to the empirical property of »impenetrability« (*Undurchdringlichkeit*), manifested by a resistance to penetration (*Eindringen*). According to the theorem, »matter fills space not by its mere *existence*, but through a *particular moving force*«. It is useful to quote Kant's »proof« (*Beweis*) of this theorem, as an example of how he moves from the empirical property of impenetrability to dynamics by means of a priori inferences.

> Das Eindringen in einen Raum (im Anfangsaugenblick heißt solches die Bestrebung einzudringen) ist eine Bewegung. Der Widerstand gegen Bewegung ist die Ursache der Verminderung, oder auch Veränderung derselben in Ruhe. Nun kann mit keiner Bewegung etwas verbunden werden, was sie vermindert oder aufhebt, als eine andere Bewegung. Also ist der Widerstand, den eine Materie in dem Raum, den sie erfüllt, allem Eindringen anderer leistet, eine Ursache der Bewegung der letzteren in entgegengesetzer Richtung (Phoron. Lehrs.). Die Ursache einer Bewegung heißt aber bewegende Kraft. Also erfüllt die Materie ihren Raum durch bewegende Kraft und nicht durch ihre bloße Existenz.[12]

Given the sensory experience of resistance to penetration (empirical element), Kant applies the phoronomical theorem of the composition of motions (mathematical element),[13] thus inferring the need to introduce a cause of this resistance, which

10 Immanuel Kant: Metaphysische Anfangsgründe der Naturwissenschaft. Riga 1786, AA IV, pp. 497f.
11 Ibid., p. 527.
12 Ibid., p. 497
13 For its proof see ibid., p. 490.

turns out to be, by definition, a moving force. [14] Kant will successively qualify this force as a fundamental repulsive force.

This argumentative context can help us understand why Kant decides to single out Lambert among those who argue for the fundamental solidity of matter. Besides Locke, who was Lambert's source, these »others« include a large number of mechanistic natural philosophers.[15] From the physical point of view, indeed, this conception corresponds to the attribution to matter of an »absolute impenetrability«, which is »nothing more nor less than an occult quality. For one asks what the cause is for the inability of matters to penetrate one another in motion, and one receives the answer: because they are impenetrable«.[16] Absolute impenetrability, together with »absolute homogeneity«, forms the basis of the »mechanical philosophy of nature«, which »under the name of *atomism*, or the *corpuscular philosophy*, always retained its authority and influence on the principles of natural science, from Democritus of old, to Descartes and even to our time«.[17] This philosophy has the advantage of representing empty spaces and vacuum in bodies with mathematical evidence, but pays this advantage with several shortcomings for mathematical physics besides its empty explanation of impenetrability (e.g., it gives too much power to the imagination in positing these empty spaces for the explanation of different densities). Kant's »dynamical mode of explanation«, on the contrary, is »much more appropriate and conducive to natural philosophy, in that it leads directly to the discovery of matter's inherent forces and laws«,[18] whereby resistance can be traced back to a cause and »estimated in regard to its degrees«.[19]

The mechanical philosophy with absolutely hard particles, empty spaces and no inherent forces could be attributed to major physicists of the time. In the first *Critique*, indeed, Kant writes that this is the way »most mathematical and mechanical investigators of nature« explain different densities.[20] These scientists include Newton and Euler, who are often discussed in the *Remarks* of the MAN. After all, one of the objectives of Kant's MAN was to provide the sketch of a new »metaphysical part« to be included in treatises of mathematical physics.[21] It is puzzling, in this regard, that Kant names Lambert in his *Remark* about solidity. As I will argue, this

14 For a detailed account of Kant's general methodology in the MAN see Paolo Pecere: La filosofia della natura in Kant. Pagina. Bari 2009, pp. 321–391; Michael Friedman: Kant's construction of nature. Oxford 2013, pp. 1–33, pp. 564–580.

15 For a list of alternatives see Konstantin Pollok: Kants »Metaphysische Anfangsgründe der Naturwissenschaft«. Ein kritischer Kommentar. Hamburg 2001, pp. 229–231.

16 Kant: Metaphysische Anfangsgründe (see fn. 10), AA IV, p. 502.

17 Ebd., p. 533.

18 Ebd., p. 533.

19 Ebd., p. 503.

20 Immanuel Kant: Critik der reinen Vernunft. Riga ~~2~~1787 (2nd ed.), AA III, p. 156.

21 Kant: Metaphysische Anfangsgründe (see fn. 10), AA IV, p. 478.

suggests that Kant is raising a *methodological* rather than a *physical* issue, concerning his new conception of a metaphysics of corporeal nature. In order to introduce this argument let us first consider Lambert's view.

3 Lambert's principles of solidity

Lambert's accounts of solidity in the *Neues Organon* and the *Architektonik* present an intertwining of empirical and rational elements, which will form the starting point of Kant's successive rethinking. In the *Neues Organon* Lambert introduces the empirical concept of matter as follows: »Der Begriff der *Materie*, den wir unmittelbar durch das *Gefühl* haben, macht, daß wir der Materie eine *Solidität* und *Festigkeit* oder *Undurchdringlichkeit* beylegen«.[22]

this text should be in a new paragraph with indentation. I used the original German only for such long, indented quotes (e.g. see next quote in this page)

He argues that, given simple concepts, which can be thought for themselves, we can immediately deduce a corresponding set of principles (*Grundsätze*). In this particular case, we get principles of »solidity« and »force«, including the principle stating that »every solid excludes any other solid from the place where it is«.[23] This suggests that the latter principle can be derived by a pure logical inference and thus be grounded on the principle of contradiction.

A similar and more detailed account can be found in the *Architectonic*. Here Lambert lists different *Grundsätze* which can be applied to »material solids« with no contradiction (*ohne Widerrede*):

> Der Begriff *Solidität* gibt uns ebenfalls einige Grundsätze, die bei dem materiellen Soliden ohne Widderrede angewandt werden.
> 1. Das Solide füllt einen Raum aus, so weit es geht.
> 2. Das Solide schließt anderes Solides von dem Orte aus, da es ist.
> 3. Das Solide hat drei Dimensionen des Raumes.
> 4. Der Raum kann mit Solidem nicht mehr als ausgefüllt sein.
> 5. Das Solide hat eine absolute Dichtigkeit, und daher ist es eine Einheit, die unveränderlich ist.[24]

In Lambert's text we find a principle of impenetrability (n. 2 above) and more striking correspondences with Kant's later account:

– These propositions can be derived by »the collection and immediate comparison [*Vergleichung*] of simple concepts«, and thus merely depend on the principle of contradiction.[25]

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22 Lambert: Neues Organon (see fn. 2), Alethiologie, § 19.
23 Ibid., § 94.
24 Lambert: Architectonic (see fn. 5), § 88.
25 Ibid., § 76.

– The term »solidity« has a number of different meanings, including metaphysical ones (»Das Wort *Solidität* ist bereits auf verschiedene Arten metaphorisch und transzendent gemacht«). It can mean »volume« in geometry, »rigidity« in physics and it can also have a metaphorical meaning regarding »scientific« or »grounded« knowledge.[26] We can even conceive of a »solidity« of spiritual substances, although we don't have any simple concept of the latter (remember Kant's observation that his principle is valid »wenigstens in der äußeren Sinnenwelt«.[27]
– The solid has »absolute density« (principle n. 5 above), a claim that will be thoroughly criticized by Kant (see below § IV).

From these close correspondences we can conclude that Kant was taking Lambert's texts in close consideration when he was writing his *Remark*. Hence we can wonder why Kant examines Lambert's *methodological* works in the MAN and why, while focusing on method, he spells out his disagreement with Lambert concerning the physical concept of *solidity* in particular.

4 Solidity from the physical point of view

against

Lambert's systematic treatment of the fundamental concepts of mechanics is better understood ~~on~~ the background of Newton's. In the *Principia mathematica* Newton considers density as an essential property of matter (hence invariable) and derives mass by multiplying density and volume. Force, on the other hand, is a variable quantity and thus no essential property.[28] Newton then introduces a cinematic measure of force, which is applied to the study of gravity, leaving open the issue of the cause of force.[29] Regarding the resistance to motion (inertial mass) he defines a *vis inertiae* of matter.[30] The ontological relevance of solidity is highlighted in *Query 31* of the *Opticks* (starting from the Latin edition of 1704), where Newton argues that God has created »solid, massy, hard, impenetrable Particles«.[31] The connections among density, mass, force and solidity require the discussion of a number of different conjectures about ether and microscopic forces, without leading to a conclu-

26 Ibid., §§ 92f.
27 Ibid., § 90.
28 Isaac Newton: Philosophiae naturalis principia mathematica, the third edition (1726) with variant readings. Ed. by Alexandre Koyré and I. Bernhard Cohen. Cambridge 1972, pp. 39f. (Definitiones, I).
29 Ibid., p. 298 (Lib. I, Sect. XI, Prop. LXIX, Scholium).
30 Ibid., p. 40 (Definitiones, III). Compare Isaac Newton: Opticks, based on the fourth edition (1730). New York 1952, p. 397 (Query 28).
31 Ibid., p. 400.

sive theory: this problematic legacy would be shared by Newtonian scientists of the 18th century.

In this context, Lambert submits an empirical foundation of mechanical concepts, arguing that both density (see above) and force depend on solidity, which is in turn given with the sensation of touch. Hence he adds a number of principles of »Dynamics« to the principles of solidity:

> Wir haben denselben, so wie den Begriff der *Solidität*, vom Gefühle, weil wir empfinden, daß wir mehr oder minder *Kraft* anwenden müssen, einen Körper oder das Solide in Bewegung zu setzen oder die Bewegung desselben zu ändern oder ganz aufzuhalten. Daraus fließen folgende Grundsätze, welche man in der Dynamik angenommen [hat].
> 1. Das Solide ist an sich in Ruhe, oder ohne Bewegung.
> 2. Das Solide wird durch anderes Solides in Bewegung gesetzt.
> 3. Jede Änderung in der Bewegung des Soliden wird durch anderes Solides verursacht, welches das bewegte Solide unmittelbar berühret.
> 4. Im freyen Raume behält das einmal in Bewegung gesetzte Solide sein Richtung und Geschwindigkeit.
> 5. Die Bewegung ist in Verhältniß [mit] der Kraft, womit das Solide in Bewegung gesetzet wird, und folget der Richtung, nach welcher die Kraft angebracht wird.[32]

To sum up: the sense of touch, by the feeling of impenetrability, is the source of both the concepts of density (and hence mass) and of inertia (the resistance to the impressed motion whose coefficient is mass – our inertial mass). These concepts are sufficient to establish the mathematical analysis of motion, while the first cause of motion is left out of the physical picture. A similar, radically mechanistic system of concepts had been recently defended by Euler: impenetrability is the source of mechanical action in impact or pressure; action is transmitted by impact or pressure according to a coefficient (inertial mass) which can be empirically measured. Force is given by multiplying mass by the impressed acceleration, while the Newtonian expression *vis inertiae* (for inertial mass) has to be avoided.[33] This approach to mechanics, in fact, would be much more successful than the dynamical one entertained by Kant and others. In this perspective, Lambert appears as just one among many mechanistic scientists following Newton and Euler.

But Lambert's account, albeit it does not envisage any repulsive force, shares an element with the Kantian account: Lambert grants that density may have an originally different degree, allowing for the explanation of specific gravity of bodies with no hypothetical dissemination of void in bodies:

> Eine andere Frage aber, die hiebey vorkömmt, ist diese: Ob ein ganz ausgefüllter Raum nicht noch *intensive* mehr ausgefüllet, oder das Solide, das ihn ausfüllet, in einen noch kleinern

format footnote

[32] Lambert: Architectonic (see fn. 5), § 94.

33 See e. g. Leonhard Euler: Mechanica sive motus scientia analytice exposita. Bd. 1. St. Petersburg 1736, §§ 56–74.

> Raum gebracht werden könne, oder ob alles Solide in sich gleich dichte und in dieser Absicht eine absolute und unveränderliche Einheit sey? Diese Fragen betreffen den zweyten, vierten und fünften Grundsatz, welche sich auf den bekannten und auch in der Mechanik angenommenen Begriff der Undurchdringlichkeit der Materie gründen. [...] Wir haben den Begriff der Solidität durch das Gefühl, und dieses giebt uns die innern Unterschiede desselben nicht an. In dem Begriffe den wir davon haben, scheint auch keine Unmöglichkeit zu liegen, daß das Solide nicht verschiedene Grade der innern Dichtigkeit haben könne.[34]

This alternative theory of density would involve a modification of Lambert's own principle n. 2, which establishes absolute density. In this perspective Lambert appears closer to Kant than the other mechanistic physicists. Indeed, this open-mindedness regarding the explanation of density casts a shadow on Lambert's claim that his axiomatization of solidity is a »grounding principle of metaphysical truth« and possesses »geometrical necessity«.[35] Michael Friedman has argued that this open-mindedness is a reason why Kant singles out Lambert in his defense of the dynamical conception, and points out that »this very open-mindedness exposes ~~deppe~~ deep problems and tensions in Lambert's overall view« on the a priori principles, concerning in particular »how such explicitly non-analytical or non-definitional a priori judgments are possible«.[36]

I agree with Friedman's suggestion that here Kant's procedure of ~~bring~~ bringing the concepts of logic »into relation with both pure and empirical intuition« determines the divergence between Kant's and Lambert's views.[37] Hence, I conclude that the reason for Kant's choice of Lambert as the first example of a supporter of solidity *cannot* lie in Lambert's physical opinions about density. Kant was rather interested in Lambert's methodological reflection on the intertwining of empirical, metaphysical and mathematical elements of physics. In order to highlight the points of agreement and disagreement regarding method we need to consider Lambert's and Kant's approach in more detail.

5 Lambert's method: the possibility of concepts and the role of examples

Lambert, in his first letter to Kant (1765), notes the »similarity« between the latter's views in the *Beweisgrund* (1763) and his own views in the *Neues Organon*.[38] Then he

34 Lambert: Architectonic (see fn. 5), § 91.
35 Ibid., § 298, 313.
36 Friedman: Kant's construction of nature (see fn. 14), p. 128.
37 Ibid., p. 129.
38 Johann Heinrich Lamberts deutscher gelehrter Briefwechsel. Hg. von Johann Bernoulli. Berlin 1782 (LPS 9), p. 335.

addresses the issue of the method of metaphysics, arguing that Wolff was mistaken in granting a fundamental role to definitions without investigating the possibility of the corresponding objects. A better model is the method followed by Euclid in geometry, where definitions are just a kind of »nomenclature«, while the possibility of the corresponding objects (geometrical figures) is merely hypothetical and has to be proved »synthetically«:

> Definitionen sind bei Euclid gleichsam nur die Nomenclatur, und der Ausdruck *per definitionem* gilt bey ihm nicht mehr als der Ausdruck *per hypothesin*. *Wolf* scheint auch nicht genug darauf gemerkt zu haben wie sorgfältig *Euclid* ist, und wie sehr er selbst die Ordnung des Vortrages dazu einrichtet, die *Möglichkeit* der Figuren zu beweisen und ihre *Gränzen* zu bestimmen. Dabey [with metaphysical concepts] muß man wohl nicht anfangen, wenn man sich nicht in einer endlosen Analysi verlieren und verwirren sondern nach Euclidens Art synthetisch gehen will.[39]

Kant replies that there was indeed »a lucky agreement in our methods«,[40] mentioning the similar role played by constructions in mathematics according to the *Deutlichkeit*. The agreement concerning metaphysics is not clearly spelled out, but Kant mentions his project to write a book on the »proper *Method of Metaphysics*«: this ambitious project is temporarily shelved because he lacks »examples, of how the correct method should work«, and he has decided to »give precedence to smaller works«. Then he mentions two »ready« writings, the »Metaphysische Anfangsgründe der natürlichen Weltweisheit« and the »metaph. Anf. der praktischen Weltweisheit«, which will present examples of the new method. By this separate exposition »the main work will not be burdened with too many and inadequate examples«.[41]

Lambert (December 3, 1766) replies that Kant's procedure of introducing the new method by the exposition of positive and negative examples is a good one, and that he has followed the same method in the *Dianoiologie*, e. g. in the illustration of the relation between form and matter of knowledge. While these concepts do not raise issues in their logical meaning, when applied in metaphysics they have led to »controversies and hypotheses«. In this regard, Lambert submits a number of propositions concerning the transition from simple to complex concepts:

> 1) Die Form giebt *Principia*, die Materie aber *Axiomata* und *Postulata*.

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39 Ibid., p. 338. That Lambert consider Euclid, rather than Wolff, as the »true auctoritas« for realizing his foundational project has been highlighted by Paola Basso: Filosofia e geometria. Lambert interprete di Euclide. Firenze 1999, in part. pp. 1f. For the proximity of Lambert's project to the Leibnizian idea of the ars characteristica see Enrico Pasini: L'altra faccia dell'uomo della Luna. Leibniz e l'»Erfindungskunst«. In: Massimo Mori, Stefano Poggi (eds.): La misura dell'uomo. Filosofia, teologia, scienza nel dibattito antropologico in Germania (1760–1915). Bologna 2005, pp. 49–70.
40 Lambert: Briefwechsel (see fn. 38), p. 341.
41 Ibid., p. 342.

> 2) Die Form fordert, daß man bey einfachen Begriffen anfange, weil diese für sich, und zwar weil sie einfach sind, keinen innern Widerspruch haben können, oder für sich davon frey und für sich gedenkbar sind.
> 3) *Axiomata* und *Postulata* kommen eigentlich nur bey einfachen Begriffen vor. Denn zusammengesetzte Begriffe sind a priori nicht für sich gedenkbar. *Die Möglichkeit der Zusammensetzung* muß erst aus den Grundsätzen und *Postulatis* folgen.[42]

In the following passage, Lambert explains in more detail this transition and the role played by axioms and postulates:

> Nach diesen Sätzen trage ich kein Bedenken zu sagen, daß Locke auf der wahren Spur gewesen, das einfache in unserer Erkenntniß aufzusuchen. Man muß nur weglassen, was der Sprachgebrauch mit einmengt [...] Der Begrif *Dauer* und eben so die Begriffe *Existenz*, *Bewegung*, *Einheit*, *Solidität*, usw haben etwas einfaches, das denselben eigen ist, und welches sich von den vielen dabey mit vorkommenden Verhältnißbegriffen sehr wohl abgesondert gedenken läßt. Sie geben auch für sich *Axiomata* und *Postulata* an, die zur wissenschaftlichen Erkenntniß den Grund legen und durchaus von gleicher Art sind, wie die Euclidischen.[43]

According to Lambert, metaphysics begins with simple concepts derived from experience. These simple concepts can be listed in any order (»in der Ordnung, wie es mir einfällt«), but it is crucial to separate their different meanings and to exclude those who can find no empirical support and are merely suggested by the corresponding word – as we have seen above with «solidity«.[44] Here Lambert follows Locke's teaching on language and definitions. Given the refined list of simple concepts the corresponding axioms and postulates can be logically derived. Axioms and postulates guide the formulation of propositions from simple concepts and hence they are the »ground« of scientific knowledge, just as it happens in geometry: on this point Lambert departs from Locke and introduces a domain of a priori inference in philosophy.

An important difference between philosophy and geometry lies in the potentially misleading form of language with regards to reference: contrary to geometry, in philosophy we face the risk of formulating judgements with a predicate which is not uniformly applied to the subject, e. g. »the watch is made of gold« when only the watch-case is made of gold. In this regard Lambert hopes that Kant's contributions will help and will be published soon. He is optimistic concerning the development of this new method of philosophy, underscoring that examples will play a crucial role in this enterprise. He then focuses on the analogy between geometrical figures and philosophical examples, arguing that

42 Ibid., p. 348 (italics are mine).
43 Ibid., p. 349.
44 Ibid.

> sich eine ächte Methode am besten und sichersten durch Vorlegung *wirklicher Beispielen* [...] mit allen Individualien zeigen kann, da sie hingegen logisch ausgedrückt leicht zu abstract bleiben würde. [...] *Beyspiele thun dabey eben den Dienst, den die Figuren in der Geometrie thun, weil auch diese eigentlich Beyspiele oder speciale Fälle sind.*[45]

The correspondence will reprise after the publication of Kant's *Dissertation* in 1770. The new exchange will focus on the reality of time, and, while Lambert will briefly point out that metaphysics can be exposed in merely abstract terms and then applied to phenomena, the role of examples will not be discussed anymore.

Yet it is interesting, in this regard, to examine Lambert's letter to Holland of November 21, 1765. In this letter Lambert comments again on Euclid's method. Euclid sets forth nominal definitions (as a »nomenclature«) and then, first, he »requires the unconditioned possibility of straight lines and circles of any magnitude and position«, and second, *ex concessis postulatis*, he shows the »universal and unconditioned possibility of the equilateral triangle«. Lambert concludes by examining the analogous procedure in the case of metaphysics, where terms are abstract and their meaning cannot be »put in front of the eyes«.

> In den Beweisen braucht Euclid den Ausdruck *per definitionem* im geringsten nicht anders als den Ausdruck *per hypothesin*. Denn bis die Möglichkeit des Begrifs nicht erwiesen ist, ist die Definition nur noch eine Hypothesis. Ist es für sich oder auch nur durch ein einziges Beyspiel klar, daß es wenigstens einige solcher Figuren giebt, die die Definition anzeigt, so mag die Definition voraus geschickt werden [...]. Die Bedingungen ihrer Möglichkeit müssen aber aus Grundsätzen und *Postulatis* folgen. Dies ist der Fall von dem Δ [...]. Dieses ist nun meines Erachtens die Art wie Euclid mit Definitionen und Begriffen umgeht. Sie solle in der Metaphysik auch angehen. Man kann aber darinn die *Sache* selbst, welche abstract ist, nicht vor Augen legen, sondern muß sich mehrentheils mit *Wort* und *Begrif* begnügen.[46]

The impact of these conceptions on Kant's parallel investigation in the 1760s is not easily assessed. In the *Deutlichkeit* (written in 1762) Kant already maintained that mathematics can show the possibility of its concepts »*in concreto*« by means of constructions, while philosophy deals with signs and cannot follow this method, thus facing the risk of using empty concepts.[47] He will hold this view of definitions in philosophy until the *Architectonic* chapter of the first *Critique*. However, one wonders what did Kant make of Lambert's ideas about the synthetic method and the role of examples in metaphysics. These ideas have probably influenced Kant's development of the unpublished material on the »metaphysical principles«, and he could have given a fresh look at Lambert's published correspondence before writing the

45 Ibid., p. 351 (italics are mine).

46 Ibid., p. 30.

47 Immanuel Kant: Untersuchung über die Deutlichkeit der Grundsätze der natürlichen Theologie und Moral (1764), AA II, pp. 278f.

MAN. As matter of fact, Lambert's ideas are closely connected to the issues addressed in the MAN.

Hence we can put forward a number of hypotheses:

– The MAN can be considered as the definitive version of the material announced by Kant in the 1760s as a »metaphysics of natural philosophy«. The book would reformulate physical doctrines which had been already anticipated in previous writings (e. g. the conceptions of fundamental forces and inertia), but first of all it would address the problem of the method of metaphysics and its epistemological importance for natural science, which is the main topic of the *Preface*. In this regard, the model first provided by Lambert would provide an important background for Kant's theory.
– The concept of solidity was the main topic of confrontation with Lambert's methodological ideas. Indeed, according to Kant, impenetrability offers the first empirical property of any physical object,[48] and Lambert similarly associates solidity with »Ding« or »etwas Reales«.[49] Moreover, the explanation of impenetrability corresponds to the dynamical theory of matter, which is, since the *Monadologia physica*, Kant's favorite example of »metaphysics combined with geometry«.[50]

In this regard, it is important to mention a major shift in Kant's thought, occurred around the time of his correspondence with Lambert. In the 1750s and early 1760s Kant grounds natural philosophy on a theory of physical monads as point-like centers of repulsive and attractive forces, and explains the originally variable density of bodies through the interplay of these forces (hence, in Kant's system of concepts, density is not a primitive magnitude anymore). ~~Now, in~~ the *Prize Essay* (written in 1762) Kant quotes the *analysis* of the concept of impenetrability as an example of metaphysical method, that leads with intuitive certainty to the action of force. Later Kant would replace physical monadology with a different dynamical theory, grounded on the representation of matter as a continuum. I cannot address here the details of this turn[51] (note that Lambert brackets the role of monads in the systems of scientific concepts. This may have provided an important example for Kant[52]). What ~~matter~~ for our purposes is that Kant, in the MAN, maintains that force (thus also density and mass) cannot be derived by analysis of concepts: the representation of motion in pure intuition involves a *synthetic* passage in both cases. As a result of

In

matters

48 Kant: Metaphysische Anfangsgründe (see fn. 10), AA IV, p. 510, quoted below, § VI.
49 Lambert: Architectonic (see fn. 5), § 57.
50 Immanuel Kant: Monadologia physica (1756), AA I, p. 473.
51 For my account see Paolo Pecere: Monadology, materialism and Newtonian forces: the turn in Kant's theory of matter. In: Quaestio 16 (2016), pp. 167–189. For a broader contextualization also see La filosofia della natura in Kant (see fn. 10), pp. 34–143.
52 Lambert: Architectonic (see fn. 5), § 90.

Kant's abandonment of monadology in the 1760s, the dynamical theory – with its methodological background – would need a complete rethinking. Let us now elaborate on these hypotheses, starting from Kant's methodical framework of the MAN.

6 Metaphysics of nature: from Lambert to Kant

Let me review the main steps of Lambert's scientific method. First, there is an empirical moment, consisting in the collection of simple concepts, derived from sensation (e. g. solidity and force from the sensation of touch). Second, by following Euclid's model, principles and postulates are derived from simple concepts. These establish the »possibility of composition«, or the »conditions of possibility« of concepts. Third, these principles and postulates are grounds of scientific propositions. E.g., from solidity we get principles of dynamics, such as the law of inertia. This inference is grounded on »logical truths«, which in turn presuppose – in a Wolffian fashion – the »metaphysical truth« of the existence of God, lest that logical truth becomes an »empty dream«. This is the also ultimate ground of the solidity of bodies:

> Demnach zieht der Satz, daß es nothwendige, ewige, unveränderliche Wahrheiten gebe, die Folge nach sich, daß ein nothwendiges, ewiges, unveränderliches *Suppositum intelligens* seyn müsse, und daß der Gegenstand dieser Wahrheiten, das will sagen, das Solide und die Kräfte, eine nothwendige Möglichkeit zu existiren haben.[53]

Of course Kant breaks with this metaphysical approach in criticism.[54] Still, he retains much of Lambert's original insights when introducing his new »metaphysics of nature«, in the MAN. Kant separates a transcendental part of this doctrine from a second more specific part.

> Sie beschäftigt sich mit einer besonderen Natur dieser oder jener Art Dinge, von denen ein empirischer Begriff gegeben ist, doch so, daß außer dem, was in diesem Begriffe liegt, kein anderes empirisches zur Erkenntnis derselben gebraucht wird (z. B. sie legt den empirischen Begriff einer Materie, oder eines denkenden Wesens zum Grunde und sucht den Umfang der Erkenntniß, deren die Vernunft über diese Gegenstände a priori fähig ist).[55]

So far, there is a close analogy with Lambert's method: Kant starts with empirical concepts – e. g. impenetrability of matter – and then derives knowledge a priori. But here we find an entirely different way of conceiving the role of mathematics, which provides not merely a methodological model, but also a necessary element of the new metaphysics. This change depends on the role of pure intuition for scientific

53 Ibid., § 299.

54 For this transition compare Cassirer: Das Erkenntnisproblem (see fn. 1), Bd. 3, p. 457.

55 Kant: Metaphysische Anfangsgründe (see fn. 10), AA IV, p. 470.

knowledge in general and the thesis that pure rational principles are necessary but insufficient for »proper natural science«, as »there can be only as much *proper* science as there is mathematics therein«.[56] This intuitive component of knowledge is necessary for establishing the *possibility* of concepts:

> Also wird, um die Möglichkeit bestimmter Naturdinge, mithin um diese a priori zu erkennen, noch erfordert, daß die dem Begriffe correspondirende *Anschauung* a priori gegeben werde, d. i. daß der Begriff construirt werde (AA IV, p. 470).

Hence Kant disagrees with Lambert's way of proving the possibility of concepts through simple empirical concepts and logical ~~inference~~. In the light of Kant's procedure, indeed, some of the principles listed by Lambert turn out to lack apodictic validity. This novel conception requires the *construction* of these properties in space, but since pure intuition of natural objects is impossible, Kant's metaphysics of bodily nature establishes the possibility of the mathematical *construction* of the properties belonging to the concept of matter.

inferences

> Damit aber die Anwendung der Mathematik auf die Körperlehre, die durch sie allein Naturwissenschaft werden kann, möglich werde, so müssen Prinzipien der *Konstruktion* der Begriffe, welche zur Möglichkeit der Materie überhaupt gehören, vorangeschickt werden; mithin wird *[1]* eine Vollständige Zergliederung des Begriffs von einer Materie überhaupt zum Grunde gelegt werde müssen, welches ein Geschäfte der reinen Philosophie ist, die zu dieser Absicht sich keiner besonderen Erfahrungen, sondern nur dessen, was sie im abgesonderten (obzwar an sich empirischen) Begriffe selbst antrifft, *[2]* in Beziehung auf die *reinen* Anschauungen im Raume und der Zeit (*[3]* nach Gesetzen, welche schon in dem Begriffe der Natur überhaupt anhängen) bedient, mithin eine wirkliche *Metaphysik der körperlichen Natur* ist.[57]

Bold, not italics

(see fn 57 below)

Here Kant summarizes the three elements of his new »metaphysics of bodily nature«, as it is exposed and applied in the MAN:

1) The analysis of the properties belonging to the *empirical* concept of matter, as the basis to seek »that sphere of cognition of which reason is capable a priori concerning these objects«.[58]

2) The schematization of these properties through motion as the »basic determination« of any material object,[59] which can be represented a priori in intuition, as a pure synthesis of space and time, thus rendering a *mathematical* science of nature possible in general.

3) The elaboration of the principles of this construction in a »pure part« of physics corresponding to these properties. This includes the *transcendental* principles of the intellect, and further *metaphysical* principles, elaborated by the intellect starting

56 Ibid., p. 470.
57 Ibid., p. 472, my numeration in bold characters.
58 Ibid., p. 470.
59 Ibid., p. 476.

from the empirical concept of matter, »that make the concept of their proper object, namely, matter, a priori suitable for application to outer experience, such as the concept of motion, the filling of space, inertia, and so on«.[60] These metaphysical principles include the explanation of the filling of space through the action of a fundamental force.

Let me examine in more detail the connection of these three elements with special regard to the example examined above in § 2 (*Dynamics*, *theorem* 1). Kant points out that the concept of impenetrability is given »by means of the sense of feeling«, which »provides us with the quantity and figure of something extended, and thus with the concept of a determinate object in space, which forms the basis of everything else one can say about this thing«.[61] The transition from the *sensation* of impenetrability to the *concept* of the filling of space requires the application of the category of quality, and the corresponding principle of intensive magnitudes (degree): thus we get the representation of the filling of space as a magnitude that can have a determinate degree. The possibility of constructing this concept, in turn, can be examined a priori in pure intuition by schematizing resistance as a motion opposing penetration. The result is one of Kant's non-pure, a priori synthetic principles of metaphysics: »matter fills space, not through its mere *existence*, but through a particular *moving force*«.

This is the proposition to which Kant's remark about Lambert is appended. We can now draw some conclusions about Kant's remark.

First, Kant's insistence on the ambiguity of the word »solidity«, while evoking Lambert's views on terminological clarity – originally meant to avoid transcendent meanings –, also implies a number of corrections to the latter's analysis of the concept. Sense experience of impenetrability, which is a first meaning of »solidity«, is not sufficient to establish a primitive concept of science. Only by the understanding of solidity as the filling of space, as a special case of intensive magnitude, does resistance to penetration become accessible to mathematical analysis. Kant's account contributes to the issue whether density is originally variable or not – a question that Lambert rightly considered unsolvable on the ground of sensation, but then left open – by developing a model of matter with originally variable density. Thus Kant's remark can be considered as both a critique and an original development of Lambert's idea that experience can provide the »occasion« for the »a priori reasoning«, based on the fundamental concept of solidity (see above § I).

But solidity is just an example – albeit a fundamental one, due to the empirical precedence of the corresponding experience. The formation of the concepts of inertia and mass, which Lambert derived as well from the experience of resistance to motion, requires the application of principles of relations in the pure part of me-

60 Ibid., p. 472.
61 Ibid., p. 510.

chanics. In the *Mechanics* chapter Kant first introduces the conditions to represent a material substance as a quantity of matter (mass) which is constant in time and manifests itself in impact and resistance to force. Here the application of the transcendental principle of causality leads to the metaphysical principle that the cause of motion must be external to the moved substance, the *lex inertiae*. Thus material substance and inertia (as lack of activity), through the combination of metaphysics and mathematics, appear as new concepts derived from the original experience of touch.

On the whole, Kant's new »principles for the construction of the concepts that belong to the possibility of matter«[62] – meant to connect a priori sense experience and mathematical physics – can be considered as a rethinking of Lambert's idea of »principles and postulates« dealing with the »possibility of composition«. The difference is that Lambert thinks to an elaboration of empirical concepts by means of logical inference which is *analogous* to mathematical construction, but does not require the use of spatial intuition, while Kant, after having defended the idea of an entirely analytical metaphysics in the *Deutlichkeit*, now introduces synthesis in pure space-time as the procedure that has to connect empirical concepts and metaphysical principles, thus establishing the possibility of mathematical constructions in physics.

Kant's view of the systematical meaning for general metaphysics of these examples concerning bodies also echoes Lambert's ideas. Remember that Lambert had repeatedly argued that his new principles would provide metaphysics with »examples«, which would play the same role of figures in Euclidean geometry, that is, to establish the possibility of defined objects. In the MAN Kant assigns a strikingly similar role to examples in metaphysics:

> Die allgemeine Metaphysik in allen Fällen, wo sie Beispiele (Anschauungen) bedarf, um ihren reinen Verstandesbegriffen Bedeutung zu verschaffen, diese jederzeit aus der allgemeinen Körperlehre, mithin von der Form und den Principien der äußeren Anschauung hernehmen müsse und, wenn diese nicht vollendet darliegen, unter lauter sinnleeren Begriffen unstät und schwankend herumptappe. Daher die bekannten Schwierigkeiten, wenigstens die Dunkelheit in den Fragen über die Möglichkeit eines Widerstreits der Realitäten, die der intensiven Größe u. a. m., bei welchen der Verstand nur durch Beispiele aus der körperlichen Natur belehrt wird, welches die Bedingungen sind, unter denen jene Begriffe allein objective Realität, d. i. Bedeutung und Wahrheit haben können. Und so thut eine abgesonderte Metaphysik der körperlichen Natur der *allgemeinen* vortreffliche und unentbehrliche Dienste, indem sie Beispiele (Fälle in concreto) herbeischafft, die Begriffe und Lehrsätze der letzteren (eigentlich der Transzendentalphilosophie) zu realisiren, d.i. einer bloßen Gedankenform Sinn und Bedeutung unterzulegen.[63]

62 Ibid., p. 472.
63 Ibid., p. 478.

This exhibition (*exhibitio*, *Darstellung*) of concepts plays a crucial role in critical philosophy in general, as Kant also points out in the new *General remark to the Analytic of principles* in the second edition of the *Critique* (AA III, pp. 200–202).[64] Again, mere sensation is a necessary but not sufficient condition of the objective reality of concepts. Therefore critique of transcendent metaphysics and foundation of natural science coincide in the new metaphysics of bodily nature. Concerning the foundational side of the project, Kant points out the epistemological failure of allegedly empiricist accounts of science by mathematical physicists:

> Alle Naturphilosophen, welche in ihrem Geschäfte mathematisch verfahren wollten, haben sich daher jederzeit (obschon sich selbst unbewußt) metaphysischer Principien bedient und bedienen müssen, wenn sie sich gleich sonst wider allen Anspruch der Metaphysik auf ihre Wissenschaft feierlich verwahrten. Ohne Zweifel verstanden sie unter der letzteren den Wahn, sich Möglichkeiten nach Belieben auszudenken und mit Begriffen zu spielen, die sich in der Anschauung vielleicht gar nicht darstellen lassen [....]. Alle wahre Metaphysik ist aus dem Wesen des Denkungsvermögens selbst genommen und keineswegs darum erdichtet, weil sie nicht von der Erfahrung entlehnt ist, sondern enthält die reinen Handlungen des Denkens, mithin Begriffe und Grundsätze a priori, welche das Mannigfaltige *empirischer Vorstellungen* allererst in die gesetzmäßige Verbindung bringt, dadurch es *empirisches Erkenntnis*, d. i. Erfahrung, werden kann. So konnten also jene mathematische Physiker metaphysischer Principien gar nicht entbehren und unter diesen auch nicht solcher, welche den Begriff ihres eigentlichen Gegenstandes, nämlich die Materie, a priori zur Anwendung auf äußere Erfahrung tauglich machen, als des Begriffs des Bewegung, der Erfüllung des Raums, der Trägheit u. s. w. Darüber aber bloß empirische Grundsätze gelten zu lassen, hielten sie mit Recht der apodiktischen Gewißheit, die sie ihren Naturgesetzen geben wolten, gar nicht gemäß, daher sie solche lieber postulirten, ohne nach ihren Quellen a priori zu forschen.[65]

This charge of inconsequence is directed – among other possible targets – to NewtOn's epistemology in the *Principia*, and Kant will indeed argue that Newton was »at variance with himself« when he denied that gravity is an essential property of matter (as Kant argues).[66] In this perspective, Lambert's approach appears once more controversial. He submitted an important insight by recognizing the importance of connecting the empirical basis of science with a priori reasoning, but his derivation of principles, lacking any reference to intuition as the source of synthetic judgments, is inadequate (in the quoted passage, indeed, Kant may have been hinting at Lambert's use of the concept of »postulate«, which corresponds to the content of Kant's new metaphysical principle).

Newton

64 I provide more details on exhibitio in my *La filosofia della natura in Kant* (see fn. 14), pp. 185–202, pp. 208–277, and: The Systematical Role of Kant's Opus postumum. »Exhibition« of Concepts and the Defense of Transcendental Philosophy. In: Con-textos Kantianos 1 (2015), pp. 156–177.

65 Kant: Metaphysische Anfangsgründe (see fn. 10). AA IV, p. 472.

66 Ibid., p. 515.

I think that Kant directly refers to this limit of Lambert's approach in the final passage of the *Remark* on solidity that we have been examining, where he addresses the »mathematician«:

> Hier hat der Mathematiker etwas als ein erstes Datum der Construction des Begriffs einer Materie, welches sich nicht weiter construiren lasse, angenommen. Nun kann er zwar von jedem beliebigen Dato seine Construction eines Begriffs anfangen, ohne sich darauf einzulassen, dieses Datum auch wiederum zu erklären; darum aber ist er doch nicht befugt, jenes für etwas aller mathematischen Construction ganz Unfähiges zu erklären, um dadurch das Zurückgehen zu den ersten Principien in der Naturwissenschaft zu hemmen.[67]

Here is the point of the methodological disagreement with Lambert: the latter considered absolute solidity as a primitive concept, thereby excluding the possibility of its construction (hence absolute impenetrability is an »empty concept«). But this impossibility, in turn, »obstructs« the natural scientist »from going back to first principles«, that is, to infer the activity of force by mathematical construction and thus to establish the objective reality of impenetrability as a consequence of natural laws. From a physical point of view, as we have seen, this produces a number of additional problems, while the dynamic philosophy envisaged by Kant reduces phenomena to laws. From the more fundamental metaphysical point of view, Lambert fails to provide an adequate account of the intertwining of logical, empirical and mathematical elements in natural science, thus also failing to justify the *necessity* of natural science, and this failure affects his treatment of the most basic empirical concept of natural science, that is, impenetrability of matter.

7 Conclusions

My examination of the disagreement between Kant and Lambert on solidity does not end up in a merely negative result. We have seen that Lambert's methodological writings not only influenced and inspired Kant in the 1760s, but – what has not been sufficiently highlighted – provided a significant legacy for Kant's methodical elaboration of a metaphysics of nature in criticism, long after the correspondence and Lambert's death. This slow elaboration is not uncommon in Kant's writings, especially ~~across~~ in his long and uninterrupted work on natural sciences. We can thus confidently conclude that Lambert's seminal insights about »protophysics« and methodology set the background for Kant's mature conception of the metaphysics of bodily nature. Among the many points of agreement we have found: the preliminary analysis of the empirical concept of matter, the role of examples against the confu-

67 Ibid., p. 498.

sions and illusory references of transcendent metaphysics, the model of Euclid's mathematical postulates for the establishment of a metaphysics of natural science. In this context, moreover, Lambert also suggests the analogy between geometrical figures and metaphysical examples, which will be developed by Kant in his theory of »principles for the construction« of matter.

Although

~~However if~~ Lambert provides the bricks and the project for the new metaphysical building, as it were, his work appears unaccomplished from Kant's point of view, because it does not spell out how empirical and a priori elements can form a properly scientific, i.e. mathematical doctrine. Lambert's conception of solidity, in particular, is an example of the failure to provide a transition from the empirical basis to mathematical physics and hence to justify the latter's exemplary function for metaphysics.

Arguably Kant may have been wrong regarding the alleged heuristic value of dynamism, as the successive history of physics shows that dynamical conceptions of matter have not provided a full reduction of the impenetrability of matter to elementary forces (or fields). Anyway, whatever we may think of Kant's solution to the problem of solidity and of its greater influence in the history of philosophy of science, it is Lambert who has to be credited for having posited this foundational problem in the framework of post-Newtonian natural sciences, thus introducing the idea of a new methodological connection between empirical, mathematical and metaphysical elements of natural science.